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We study the process of vacuum decay in quantum field theory focusing on the stochastic aspects of the interaction between long and short wavelength modes. This interaction results in a diffusive behavior of the reduced Wigner function describing the state of the long wavelength modes, and thereby to a finite activation rate even at zero temperature. This effect makes a substantial contribution to the total decay rate.

In this paper, we shall elaborate on the seemingly naive observation that field theories are systems with an infinite number of degrees of freedom, only a few of which are accessible in any given observational context. As a consequence, any application of quantum field theory implies the *a priori* selection of part of the theory as relevant, with the rest being regarded as just an environment for the relevant part [1]. From this point on, everything we say about field theory has a statistical import, and in particular, we are confronted with the field theoretical equivalents of statistical concepts such as dissipation, noise, entropy, etc. Pursuing this program, we have been concerned before with the stochastic aspects of semiclassical gravity [2–4] and effective theories [5]. The goal of this paper is to apply this perspective of quantum fields as *de facto* open systems [6,7] to analyze vacuum decay in scalar field theory [8].

As a matter of fact, two of the present authors have already considered the relevance of stochasticity in the context of the *creation from nothing* of the Universe [9,10]. Our analysis of that problem led to the conclusion that the noise induced transition amplitude was actually larger than the usual quantum estimates [11]. However, it remained unclear whether the relevance of stochasticity for the full decay amplitude was a peculiarity of gravitationally bound systems, or rather a generic feature of vacuum decay in field theory. The results we shall discuss here point quite conclusively in the second direction. In pursuit of clarity, we shall omit most of the technical details, which shall be reported in separate publications [12,13].

As a simple non - gravitational example, let us consider a self interacting scalar field Φ in Minkowski space time. The classical action is

$$S_{ren}[\Phi] = \int d^4x \left\{ \left(\frac{-1}{2} \right) [\partial\Phi^2 + M^2\Phi^2] + \frac{1}{6}g\Phi^3 \right\} \quad (1a)$$

Although we keep \hbar explicit, we set $c = 1$. M has units of $length^{-1}$, Φ has units of $M\sqrt{\hbar}$ and g of $M/\sqrt{\hbar}$. For simplicity, we shall assume that renormalization has been carried out already and that eq. (1a) is a good description of the relevant dynamics. This means that the "constants" M^2 and g may well be temperature and or renormalization point dependent; in any case, any such dependence will be taken as given [14].

We are concerned with situations where the potential displays a local minimum, separated from the absolute minimum by a potential barrier. A system of few degrees of freedom, prepared in a false vacuum state within a potential well, may decay in essentially two different ways, namely

- a) by tunnel effect, that is, going through the barrier in a classically forbidden trajectory [15–17], or else,
- b) by activation, that is, jumping above the barrier [18,19].

In systems with few degrees of freedom, there must be an external agent, typically a thermal source, for activation to be possible. Activation results from the system being driven by noise originating in the source.

In either case, the decay probability follows the Arrhenius law $P \sim Ae^{-B}$. In the tunnel effect, $B = S/\hbar$, where \hbar is Planck's constant and S is the action for the trajectory which goes under the barrier in imaginary time [20]. In activation, $B = V_s/k_B T$, where k_B is Boltzmann's constant, T is the temperature, and V_s is the height of the free energy barrier measured from the false vacuum [21,22]. We see that activation disappears as $T \rightarrow 0$.

Our thesis is that in field theories there is a phenomenon alike activation, even in the absence of an external environment. This simulated activation contributes to vacuum decay probability even at absolute zero.

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This phenomenon exists because, while vacuum decay concerns mainly the long wavelength modes in the field, these modes evolve in the environment provided by the short wavelength ones. Unlike in Kramers' activation, this environment is intrinsic to the system. We should point out that because of this same reason, we are not allowed to prescribe the characteristics of noise and dissipation independently of system dynamics. This means, it is not possible in general to assume ohmic dissipation or white noise [23,24].

Let us return to the scalar field theory above. The potential

$$V[\phi] = \frac{1}{2}M^2\phi^2 - \frac{1}{6}g\phi^3 \quad (2)$$

has a (stable) fixed point at $\phi = 0$ and an unstable fixed point at $\phi = \phi_s = 2g^{-1}M^2$. The former corresponds to zero energy, and the later to $E = E_s = VM^2\phi_s^2/6$. For intermediate energies, we may have bound and unbound states. They are separated by a potential barrier, which at zero energy extends from $\phi = 0$ to $\phi = \phi_{exit} = 3\phi_s/2$. At any given energy there will be three classical turning points $\phi_L < 0 < \phi_R < \phi_s < \phi_X$; as $E \rightarrow 0$, $\phi_L, \phi_R \rightarrow 0$ and $\phi_X \rightarrow \phi_{exit}$, while when $E \rightarrow E_s$, $\phi_R, \phi_X \rightarrow \phi_s$ and $\phi_L \rightarrow -\phi_s/2$. (see Fig. 1).

To identify the relevant modes, we observe that, if we consider fluctuations around the unstable fixed point ϕ_s , then modes with wavenumber $k > M$ are stable. The relevant modes, which partake in the tunneling process, have $k < M$ [25]. We therefore write the field as $\Phi = \phi + \varphi$, where the first term contains only modes with $k < M$, and the second term contains the short wavelengths; ϕ shall be our system.

In other words, the field ϕ represents the average of the full field Φ over volumes of order M^{-3} . By construction, ϕ is slowly varying; it is technically simplest to handle it as if it were actually spatially homogeneous. In the following, we shall adopt this approximation [26]; more sophisticated techniques to describe the system field are also available [27]. We shall therefore describe the system by a single degree of freedom $\phi(t)$, representing the field amplitude within a domain of size $1/M$.

For weak coupling, the amplitude for each mode in the environment obeys a linear equation

$$\frac{\partial^2 \varphi_k}{\partial t^2} + [k^2 + M^2 - g\phi(t)] \varphi_k = 0, \quad (3)$$

from where it is clear that modes with $k^2 > M^2$ are stable when $\phi(t) = \phi_s$. Due to the time dependence of the homogeneous mode, even if the inhomogeneous modes were initially prepared in their vacuum states, these will evolve into coherent superpositions of many particle states [28-30,2,5]. The energy to create these particles is provided by the homogeneous mode.

On the other hand, it is not possible to predict the exact number of particles to be created from the homogeneous mode. For Bose-Einstein statistics, for example, if N particles are created in the mean, then the dispersion in this number is of order $\sqrt{N(N+1)}$, and it is never negligible.

Therefore, we find a dissipative term in the dynamics of the homogeneous mode, representing the energy transfer towards the inhomogeneous modes, but also a stochastic element, related to the fluctuations in the energy flux [31]. These two terms are related to each other through the fluctuation - dissipation theorems [32]. We must stress that the presence of one of them implies the presence of the other as well.

If the quantum state of the full field is described by a density matrix $\rho(\phi, \varphi, \phi', \varphi', t)$, the state of the ϕ field is described by the reduced density matrix

$$\rho_r(\phi, \phi', t) = \int d\varphi \rho(\phi, \varphi, \phi', \varphi, t) \quad (4)$$

Or equivalently by the reduced Wigner function

$$f(\phi, p, t) = \int \frac{du}{2\pi\hbar} e^{-ipu/\hbar} \rho_r\left(\phi + \frac{u}{2}, \phi - \frac{u}{2}, t\right) \quad (5)$$

To second order in g , f may be represented as [12]

$$f(\phi, p, t) = \int D\xi P[\xi] \int d\phi_i dp_i f(\phi_i, p_i, 0) \delta(\phi - \phi(t)) \delta(p - p(t)) \quad (6)$$

where $\phi(t)$ and $p(t)$ are the solutions to the system

$$\frac{d\phi}{dt}(t) = p(t) \quad (7)$$

$$\frac{dp}{dt}(t) = -V'[\phi(t)] + \int dt' D(t-t') \phi(t') + \xi \quad (8)$$

with initial conditions (ϕ_i, p_i) , and

$$P[\xi] \cong \exp \left\{ -\frac{1}{2} \int dt dt' \xi(t) N(t-t') \xi(t') \right\} \quad (9)$$

The kernels D and N represent the effects of dissipation and noise, respectively. They are the same coefficients that appear in the quadratic part of the closed time - path (or Schwinger - Keldysh) effective action, and have been computed many times in the literature [33]

The integral representation of the reduced Wigner function is equivalent to the Kramers' equation [12]

$$\frac{\partial f}{\partial t} = \{H, f\} + \frac{\partial}{\partial P} [\Gamma f + \{N, f\}], \quad (10)$$

where the brackets are Poisson's, and

$$\Gamma = \int dt' D(t-t') \phi(t') \quad (11)$$

$$N = \int dt' N(t-t') \phi(t') \quad (12)$$

To be rigorous, we must note that eq. (10) is the proper leading equation only at finite temperature. In our problem, where the dissipative mechanism is particle creation from the vacuum, Γ and N are themselves quantum corrections, formally of the same order as the quantum corrections coming from the self interaction of the homogeneous mode. Therefore it is more accurate to write down

$$\frac{\partial f}{\partial t} = \{H, f\} + \frac{\partial}{\partial P} [\Gamma f + \{N, f\}] - \frac{g\hbar^2}{24} \frac{\partial^3}{\partial P^3} f \quad (13)$$

as transport equation up to quadratic order in g .

Our approach to eq. (13) shall be pragmatic. It is clear that if only the first term of the right hand side is kept, the equation reduces to the classical transport equation and there is no tunneling. Retaining the first and last term in the right hand side is equivalent to writing a Schrödinger equation for the wave function of the homogeneous mode, as if it were a closed system. Since this is a one dimensional problem, the tunneling rate may be computed either by the instanton or the WKB method, which are known to be equivalent in this case (see below). We wish to know if the middle term makes a substantial contribution to the total rate. With this strategy in mind, rather than more formal considerations, we shall discard the third contribution to the right hand side in eq. (13), which then reduces back to (10)

To first order in \hbar , we must use solutions to the classical equations of motion within the integrals; these solutions may be written down explicitly in terms of elliptic functions [34]. We also neglect transient terms (or in other words, we assume $t \gg M^{-1}$); this means we can take the lower limit of the time integrals in eqs. (12) as $t = -\infty$. In the weak dissipation limit, as discussed by Kramers [18], the reduced Wigner function depends only on the action variable J [35]

$$J = \frac{1}{2\pi} \oint d\phi p \quad (14)$$

(see Fig. 2) and, averaging over angles, Kramers' equation reduces to Fokker - Planck's

$$\frac{\partial f}{\partial t} + \frac{\partial \Phi}{\partial J} = 0 \quad (15)$$

where

$$\Phi = - \left\{ \frac{\Theta}{\Omega} \frac{\partial f}{\partial J} + \Lambda f \right\} \quad (16)$$

is the flux, and $\Omega = \Omega(J)$ is the frequency of the corresponding classical motion [36,37].

The point of this analysis is that it is possible to derive explicit expressions for the coefficients Θ and Λ in the Fokker - Planck equation [9,13]. Near the top of the barrier, they assume finite values, while at low energy they go to zero as E^3 . In the thermal activation problem one finds an identical equation, but the coefficients decay linearly on E [18].

The weakness of noise and dissipation in our (vacuum decay) problem reflects the origin of these effects in particle creation. Since particles are created in pairs, there is a threshold for particle creation at frequency $\omega \sim 2M$. At low energy, classical motion is mainly harmonic at the small oscillations frequency M , hence particle creation is weak. It never actually vanishes, though, because at any finite energy there is a small deviation from harmonic motion. The amplitude of the component with frequency $n\Omega$ decays as E^n as $E \rightarrow 0$, which is enough to trigger particle creation [5,38].

The boundary conditions for the Fokker - Planck equation are vanishing flux $\Phi = 0$ at the false vacuum $J = 0$, and vanishing probability $f = 0$ of finding a particle on top of the barrier at $\phi = \phi_s$. The operator $L = \partial\Phi/\partial J$ is self adjoint with respect to an adequate inner product [13], and the equation may be solved by an expansion in normal modes in the usual way [39]. A general solution is reconstructed as a superposition of modes f_r decaying as $\exp(-rt)$. For a given r , f_r oscillates as $J \rightarrow 0$, and the modes must be subject to a continuum normalization, as in the usual treatment of the WKB wave function in quantum mechanics [16]. The result is that, given any smooth initial condition with mean energies of the order of the false vacuum energy $\hbar M/2$, the persistency amplitude

$$P(t) = 2\pi \int dJ f(J, t) \quad (17)$$

decays exponentially with a constant λ for $\lambda t \gtrsim 1$, turning to $1/t$ for longer times (this crossover is also observed in the tunneling amplitude [40]). The constant is [13]

$$\lambda \approx \Delta \exp \left\{ - \int dE \frac{\Lambda}{\Theta}(E) \right\} \sim \Delta \exp \left\{ -a \frac{M^2}{\hbar g^2} \right\} \quad (18)$$

where Δ is of order 1, and $a \sim 0.2\dots$. By contrast, the tunneling amplitude, in the corresponding approximation of only considering the homogeneous mode, yields a similar formula, but with $a \sim 4.8\dots$ [20]

We see that, in this case, the zero temperature activation rate is higher than the tunneling amplitude by an order of magnitude in the exponent.

In conclusion, we have shown that vacuum decay in field theory is qualitatively different from the same process in systems with few degrees of freedom, because the former are intrinsically open systems. Interaction between long and short wavelength modes induce a stochastic dynamics for the former and results in activation even at zero temperature.

More concretely, we should point out that the fact that in our example the activation amplitude is actually larger than the tunneling amplitude is model dependent. Roughly speaking, low and broad barriers favor activation, while high and narrow barriers favor tunneling. It is safe to conclude, however, that activation should not be discarded *a priori*, but rather counted on as a potentially significant contribution to the overall decay amplitude.

We should mention that, although the vacuum decay is driven by noise from the environmental fluctuations, there is no net work done on the system by the environment, and thus the energy of the zero point fluctuations in the short wavelength modes is conserved. This is due to the compensation (in the mean) of the energy transfers due to noise and dissipation (details in [13]).

It is also important to keep in mind that we make no claims regarding their individual observability of the solutions to the Langevin equations (7) and (8). Only the reduced Wigner function f has a direct physical meaning, and it will not allow an interpretation as a classical distribution function in general, since it will not be generally positive definite. Our only claim is that the Kramers equation is the right quantum evolution equation, to lowest nontrivial order in \hbar and g .

Our results tend to disagree with the often quoted result that dissipation suppresses tunneling. While sometimes ref. [41] is cited as supporting that position, we must mention that the model analyzed there, once all relevant approximations are considered, does not comply with the fluctuation - dissipation relations, and so their results cannot be simply compared to ours. Our analysis is closer in philosophy to Bak and Bruinsma's [42], and certainly compatible with their conclusions.

Still, we must stress that we should not expect a similar behavior in systems with few degrees of freedom. The fact that in our problem the environment actually contained a large enough number of degrees of freedom as to represent a continuum for all practical purposes is essential to provide a suitable driving force. If some frequency intervals were lacking, then there would arise islands of stability where no resonance is strong enough to move the system forward. These islands would act as absolute barriers to noise induced decay, or at least would depress the noise induced amplitude much below the tunneling estimates. The similarity of this picture with the role of Kolmogorov - Arnold - Moser tori in chaotic diffusion is tantalizing [43].

We should stress that the methods we have used do not assume equilibrium conditions, and therefore they may be applied to compute transition rates in cosmological phase transitions [44,45] and other dynamical situations [46,47]. Understanding vacuum decay in strongly nonequilibrium conditions is a daunting challenge, with important applications to cosmology and high energy physics

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I. FIGURE CAPTIONS

Fig. 1: The potential, as a function of $x = \phi/\phi_s$.

Fig. 2: Classical phase space, in coordinates $x = \phi/\phi_s$, $p = \dot{\phi}/\phi_s$. The action variable J corresponds to the area enclosed by each trajectory, in units of 2π .